

--	--	--	--	--	--	--	--	--	--

# MULTIMEDIA UNIVERSITY

## FINAL EXAMINATION

TRIMESTER 1, 2015/2016

### ECT1026 – FIELD THEORY

(All sections / groups)

8 OCTOBER 2015  
2:30 p.m. – 4:30 p.m.  
(2 Hours)

---

#### INSTRUCTIONS TO STUDENT

1. This question paper consists of eight (8) pages including this cover page with four (4) questions only.
2. Attempt all **FOUR (4)** questions. The distribution of the marks for each question is given.
3. Please write all your answers clearly in the answer booklet provided.

**Question 1**

a) Magnetic circuits can be analyzed (by analogy) using similar techniques as in electrical circuits. Write down the analogous electrical quantities of the following magnetic quantities:

- |  |          |
|--|----------|
| i) Magneto-motive force, $\mathcal{F}$ . | [1 mark] |
| ii) Magnetic flux, $\Phi$ .              | [1 mark] |
| iii) Reluctance, $\mathcal{R}$ .         | [1 mark] |

b) Give three differences between electric and magnetic circuits. [3 marks]

c) Briefly explain the phenomenon of *eddy* current loss and the method to reduce it. [3 marks]

d) In the iron core shown in Figure Q1(d), the coil  $F_1$  is supplying 1000 AT in the direction indicated. The relative permeability of iron may be taken as 2500. Neglect fringing effect and flux leakage.

- |  |           |
|--|-----------|
| i) Draw the equivalent circuit of the iron core.   | [4 marks] |
| ii) Calculate reluctance $R_{bafg}$ , $R_{be}$ and $R_{bcde}$ .  | [5 marks] |
| iii) Find the mmf of coil $F_2$ and its current direction to produce an air-gap flux $\Phi$ of 5 mWb in the direction shown. | [7 marks] |

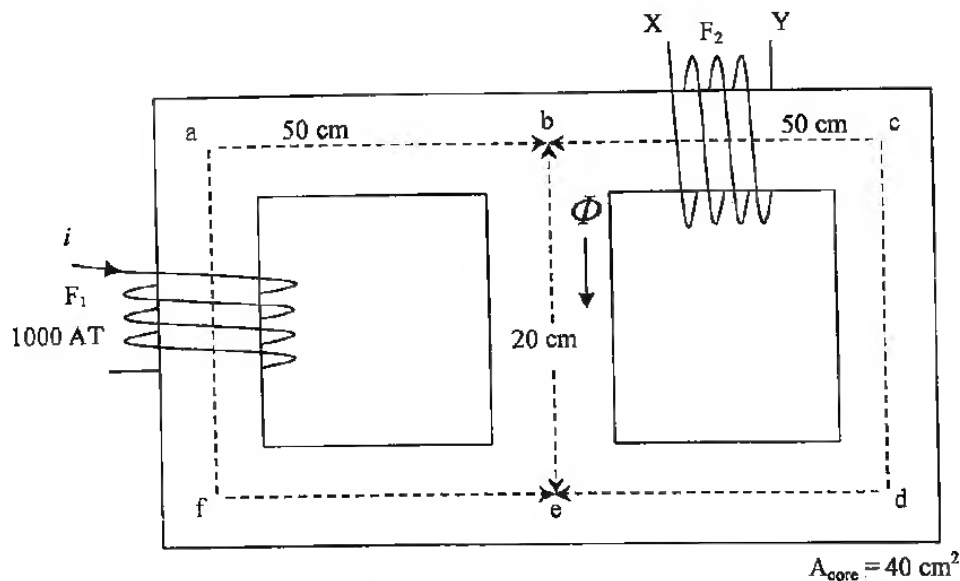


Figure Q1(d) Magnetic circuit

Continued ...

**Question 2**

a) A vector in spherical coordinate system is given by  $\vec{E} = \frac{25}{R^2} \vec{a}_R$ . It is acting on the  $x$ - $y$  plane ( $z = 0$ ) as shown below:

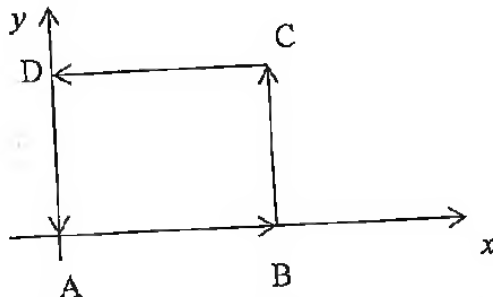


Figure Q2(a) A square on the  $x$ - $y$  plane ( $z = 0$ )

Points  $A(0,0,0)$ ,  $B(1,0,0)$ ,  $C(1,1,0)$  and  $D(0,1,0)$  form a square in anti-clockwise direction as shown in Figure Q2(a).

- i) Express the spherical vector,  $R \vec{a}_R$  in Cartesian coordinate system involving  $x$ ,  $y$ ,  $z$  and their unit vectors. Then express  $\vec{a}_R$  in Cartesian system. [3 marks]
- ii) Find  $\vec{E}$  in Cartesian system at a point with Cartesian coordinates of  $(1,2,3)$ . [4 marks]
- iii) Express  $\vec{E}$  in Cartesian system for path A to B. [2 marks]
- iv) Express  $\vec{E}$  in Cartesian system for path B to C. [2 marks]
- v) Express  $\vec{E}$  in Cartesian system for path C to D. [1 mark]
- vi) The close path integral of  $\vec{E}$  around ABCD in anti-clockwise direction is given by  $\oint \vec{E} \cdot d\vec{l}$ . Find its value in Cartesian system. [2 marks]
- vii) When  $\vec{E}$  is electric field with units of V/m, elaborate on the result you obtained in part (vi) above. What is the name of this result in circuit theory? [3 marks]

b) A vector is given as  $\vec{B} = (y^2 + 2z)\vec{a}_x + \left(\frac{\cos z}{x^2 + y}\right)\vec{a}_y + \ln(x^2 + y)\vec{a}_z$ .

- i) Find the  $\vec{a}_x$  component of the vector  $\vec{\nabla} \times \vec{B}$ . [4 marks]
- ii) Find the  $\vec{a}_y$  component of the vector  $\vec{\nabla} \times \vec{B}$ . [4 marks]

Continued ...

**Question 3**

a) A conductor is located from  $-L/2$  to  $L/2$  along the y-axis and it carries a current  $I$ . Point  $P$  is located at Cartesian coordinate system of  $(x, 0, z)$ .

- i) Sketch the conductor and point  $P$ . [3 marks]
- ii) Express the current element  $d\vec{l}$  in Cartesian system. [2 marks]
- iii) Express distant vector from  $d\vec{l}$  to  $P$ ,  $\vec{R}$ , in Cartesian system. [2 marks]
- iv) Express the vector  $d\vec{l} \times \vec{R}$  in Cartesian system. [2 marks]
- v) Find the magnetic flux density,  $\vec{B}$  at point  $P$  due to the conductor. [3 marks]

b) A magnetic material with a permeability of  $\mu_1$  is interfacing with air. When a magnetic field intensity vector,  $H_0$  passes through this interface, the following is obtained:

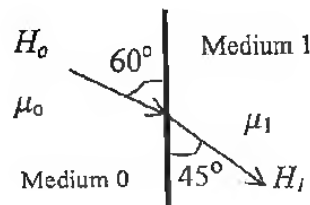


Figure Q3(b) The magnetic field intensity is deflected across the boundary

Find the ratio of  $\mu_1/\mu_0$ . [6 marks]

c) A copper wire is made into a circle of radius  $r_0$  with  $N$  turns.

- i) Find the magnetic flux density,  $\vec{B}$  at the centre of the circle. [4 marks]
- ii) Find the magnetic energy density,  $\omega_m$  at the centre of the circle. [3 marks]

Continued ...

**Question 4**

a) A positive point charge,  $+q$ , is located at (2,2) on the  $x$ - $y$  plane. The  $x$ -axis is grounded at 0V.  $A$  is a point with Cartesian coordinates of (3,3).

- i) Use image method, sketch the two charges and the two distant vectors to point  $A$ . [3 marks]
- ii)  $\vec{r}_1$  is the distant vector from (2,2) to (3,3). Find its unit vector. [2 marks]
- iii)  $\vec{r}_2$  is the distant vector from (2,-2) to (3,3). Find its unit vector. [2 marks]
- iv) Find the total electric field at point  $A$ ,  $\vec{E}$ . [4 marks]

b) A spherical capacitor is constructed using two spheres separated by a dielectric medium. The inner sphere has a radius of  $R_i$  while the outer sphere has a radius of  $R_o$ . The permittivity of the dielectric is  $\epsilon$ .

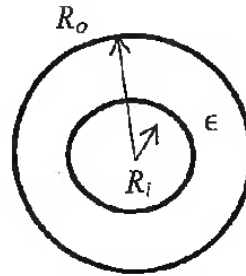


Figure Q4(b) Spherical capacitor with 2 spheres

- i) Find the electric field in the dielectric medium,  $\vec{E}$  when a total of charge  $+Q$  is deposited on the inner sphere. [4 marks]
  - ii) Find the capacitance of this structure. [6 marks]
- c) What is the meaning of 'statics' in electrostatics? [2 marks]
- d) Express the two equations from the Maxwell's Equation related to electric charges under magnetostatics condition. [2 marks]

**End of Paper**

**Appendix****Physical Constants, Vector and Coordinate Transformation Relations**

Elementary charge	$e$	$1.60 \times 10^{-19} \text{ C}$
Permittivity constant	$\epsilon_0$	$8.85 \times 10^{-12} \text{ F/m}$
Permeability constant	$\mu_0$	$1.26 \times 10^{-6} \text{ H/m}$

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Differential length	$\hat{x}dx + \hat{y}dy + \hat{z}dz$	$\hat{r}dr + \hat{\phi}r d\phi + \hat{z}dz$	$\hat{R}dR + \hat{\theta}R d\theta + \hat{\phi}R \sin \theta d\phi$
Differential surface areas	$d\vec{s}_x = \hat{x}dydz$ $d\vec{s}_y = \hat{y}dxdz$ $d\vec{s}_z = \hat{z}dxdy$	$d\vec{s}_r = \hat{r}rd\phi dz$ $d\vec{s}_\phi = \hat{\phi}rdr dz$ $d\vec{s}_z = \hat{z}rdr d\phi$	$d\vec{s}_R = \hat{R}R^2 \sin \theta d\theta d\phi$ $d\vec{s}_\theta = \hat{\theta}R \sin \theta dR d\phi$ $d\vec{s}_\phi = \hat{\phi}R dR d\theta$
Differential volume	$dxdydz$	$rdrd\phi dz$	$R^2 \sin \theta dR d\theta d\phi$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
1.	$r = \sqrt{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{r} = \hat{x} \cos \phi + \hat{y} \sin \phi$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$ $\hat{z} = \hat{z}$	$A_r = A_x \cos \phi + A_y \sin \phi$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$ $A_z = A_z$
2.	$x = r \cos \phi$ $y = r \sin \phi$ $z = z$	$\hat{x} = \hat{r} \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{r} \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{z}$	$A_x = A_r \cos \phi - A_\phi \sin \phi$ $A_y = A_r \sin \phi + A_\phi \cos \phi$ $A_z = A_z$
3.	$R = \sqrt{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}(\sqrt{x^2 + y^2}/z)$ $\phi = \tan^{-1}(y/x)$	$\hat{R} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta$ $\hat{\theta} = \hat{x} \cos \theta \cos \phi + \hat{y} \cos \theta \sin \phi - \hat{z} \sin \theta$ $\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$	$A_R = A_x \sin \theta \cos \phi + A_y \sin \theta \sin \phi + A_z \cos \theta$ $A_\theta = A_x \cos \theta \cos \phi + A_y \cos \theta \sin \phi - A_z \sin \theta$ $A_\phi = -A_x \sin \phi + A_y \cos \phi$
4.	$x = R \sin \theta \cos \phi$ $y = R \sin \theta \sin \phi$ $z = R \cos \theta$	$\hat{x} = \hat{R} \sin \theta \cos \phi + \hat{\theta} \cos \theta \cos \phi - \hat{\phi} \sin \phi$ $\hat{y} = \hat{R} \sin \theta \sin \phi + \hat{\theta} \cos \theta \sin \phi + \hat{\phi} \cos \phi$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_x = A_R \sin \theta \cos \phi + A_\theta \cos \theta \cos \phi - A_\phi \sin \phi$ $A_y = A_R \sin \theta \sin \phi + A_\theta \cos \theta \sin \phi + A_\phi \cos \phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$
5.	$R = \sqrt{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{R} = \hat{r} \sin \theta + \hat{z} \cos \theta$ $\hat{\theta} = \hat{r} \cos \theta - \hat{z} \sin \theta$ $\hat{\phi} = \hat{\phi}$	$A_R = A_r \sin \theta + A_z \cos \theta$ $A_\theta = A_r \cos \theta - A_z \sin \theta$ $A_\phi = A_\phi$
6.	$r = R \sin \theta$ $\phi = \phi$ $z = R \cos \theta$	$\hat{r} = \hat{R} \sin \theta + \hat{\theta} \cos \theta$ $\hat{\phi} = \hat{\phi}$ $\hat{z} = \hat{R} \cos \theta - \hat{\theta} \sin \theta$	$A_r = A_R \sin \theta + A_\theta \cos \theta$ $A_\phi = A_\phi$ $A_z = A_R \cos \theta - A_\theta \sin \theta$

$$(\hat{x} = \vec{a}_x, \hat{R} = \vec{a}_R, \hat{\theta} = \vec{a}_\theta)$$

## Gradient, Divergence, Curl and Laplacian Operators

Cartesian coordinate (x, y, z)

$$\nabla V = \hat{x} \frac{\partial V}{\partial x} + \hat{y} \frac{\partial V}{\partial y} + \hat{z} \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{x} \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{y} \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{z} \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical coordinate (r,  $\phi$ , z)

$$\nabla V = \hat{r} \frac{\partial V}{\partial r} + \hat{\phi} \frac{1}{r} \frac{\partial V}{\partial \phi} + \hat{z} \frac{\partial V}{\partial z}$$

$$\nabla \cdot \vec{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{1}{r} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \frac{1}{r} \begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & r A_\phi & A_z \end{vmatrix} = \hat{r} \left( \frac{1}{r} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{\phi} \left( \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \hat{z} \frac{1}{r} \left[ \frac{\partial}{\partial r} (r A_\phi) - \frac{\partial A_r}{\partial \phi} \right]$$

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical coordinate (R,  $\theta$ ,  $\phi$ )

$$\nabla V = \hat{R} \frac{\partial V}{\partial R} + \hat{\theta} \frac{1}{R} \frac{\partial V}{\partial \theta} + \hat{\phi} \frac{1}{R \sin \theta} \frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \vec{A} = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 A_R) + \frac{1}{R \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{R \sin \theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\begin{aligned} \nabla \times \vec{A} &= \frac{1}{R^2 \sin \theta} \begin{vmatrix} \hat{R} & \hat{\theta} & \hat{\phi} \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & R A_\theta & (R \sin \theta) A_\phi \end{vmatrix} \\ &= \hat{R} \frac{1}{R \sin \theta} \left[ \frac{\partial}{\partial \theta} (A_\phi \sin \theta) - \frac{\partial A_\theta}{\partial \phi} \right] + \hat{\theta} \frac{1}{R} \left[ \frac{1}{\sin \theta} \frac{\partial A_R}{\partial \phi} - \frac{\partial}{\partial R} (R A_\phi) \right] + \hat{\phi} \frac{1}{R} \left[ \frac{\partial}{\partial R} (R A_\theta) - \frac{\partial A_R}{\partial \theta} \right] \end{aligned}$$

$$\nabla^2 V = \frac{1}{R^2} \frac{\partial}{\partial R} \left( R^2 \frac{\partial V}{\partial R} \right) + \frac{1}{R^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{R^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}$$

**Table of Integrals**

$$\int \sin^2 \theta d\theta = \frac{\theta}{2} - \frac{1}{4} \sin 2\theta + \text{constant}$$

$$\int \cos^2 \theta d\theta = \frac{\theta}{2} + \frac{1}{4} \sin 2\theta + \text{constant}$$

$$\int \sin \theta \cos^2 \theta d\theta = -\frac{1}{3} \cos^3 \theta + \text{constant}$$

$$\int \cos \theta \sin^4 \theta d\theta = \frac{1}{5} \sin^5 \theta + \text{constant}$$

$$\int \sin 2\theta d\theta = -\frac{1}{2} \cos 2\theta + \text{constant}$$

$$\int x \sqrt{a^2 - x^2} dx = -\frac{1}{3} (a^2 - x^2)^{3/2} + \text{constant}$$

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \tan^{-1} \left( \frac{x}{\sqrt{a^2 - x^2}} \right) + \text{constant}$$

$$\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2} + \text{constant}$$

$$\int \frac{dz}{(r^2 + z^2)^{3/2}} = \frac{z}{r^2 \sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{r dr}{(r^2 + z^2)^{3/2}} = \frac{-1}{\sqrt{r^2 + z^2}} + \text{constant}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + \text{constant}$$

**Table of formula**

Biot-Savart's Law:  $d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{R}}{R^3} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{a}_R}{R^2}$

Ampere's Law:  $\oint \vec{H} \cdot d\vec{l} = I_{\text{enclosed}}$  ; Magnetic energy density,  $\omega_m = \frac{1}{2} \vec{H} \cdot \vec{B}$

Coulomb's Law:  $d\vec{E} = \frac{dq}{4\pi\epsilon_0} \frac{\vec{a}_R}{R^2}$  ; Electric field:  $\vec{E} = -\vec{\nabla}V$

Gauss's Law:  $\oint \vec{E} \cdot d\vec{s} = Q_{\text{enclosed}}/\epsilon$  ; Electric energy density,  $\omega_e = \frac{1}{2} \vec{D} \cdot \vec{E}$

Maxwell's Equations:  $\vec{\nabla} \cdot \vec{D} = \rho_v$  ;  $\vec{\nabla} \cdot \vec{B} = 0$  ;  $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$  ;  $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Lorentz's Equation:  $\vec{F} = q\vec{E} + q\vec{u} \times \vec{B}$

Reluctance:  $\mathcal{R} = \frac{L}{\mu A}$

**End of Appendix**